

Lesson 20

Find the *second partial derivatives* of the following functions:

$$1. f(x, y) = y^5 - 3xy \qquad f_x = -3y, f_y = 5y^4 - 3x$$

$$2. f(u, v) = u^4v^3 + 8u^2v \qquad f_u = 4u^3v^3 + 16uv, f_v = 3u^4v^2 + 8u^2$$

$$3. f(x, y) = \ln(\sqrt{x^2 + xy - 1}) \qquad f_x = \frac{2x + y}{2(x^2 + xy - 1)}, f_y = \frac{x}{2(x^2 + xy - 1)}$$

$$4. f(x, y) = \sin(xy) \qquad f_x = y \cos(xy), f_y = x \cos(xy)$$

$$5. f(x, y) = ye^{2x+3y} \qquad f_x = 2ye^{2x+3y}, f_y = (1 + 3y)e^{2x+3y}$$

$$6. f(x, y) = \frac{4x}{y} \qquad f_x = \frac{4}{y}, f_y = \frac{-4x}{y^2}$$

$$7. f(x, y) = \frac{x - y}{3x + y} \qquad f_x = \frac{4y}{(3x + y)^2}, f_y = \frac{-4x}{(3x + y)^2}$$

$$8. f(x, y) = y^2 \ln(x^2 + y^2) \qquad f_x = \frac{2xy^2}{x^2 + y^2}, f_y = 2y \ln(x^2 + y^2) + \frac{2y^3}{x^2 + y^2}$$

$$9. f(u, v) = 5uv + 5e^{uv+2v} \qquad f_u = 5v + 5ve^{uv+2v}, f_v = 5u + 5(u + 2)e^{uv+2v}$$

Answers:

#	f_{xx} or f_{uu}	f_{yy} or f_{vv}	f_{xy} or f_{uv}
1	0	$20y^3$	-3
2	$12u^2v^3 + 16v$	$6u^4v$	$12u^3v^2 + 16u$
3	$\frac{-(2x^2 + 2xy + y^2 + 2)}{2(x^2 + xy - 1)^2}$	$\frac{-x^2}{2(x^2 + xy - 1)^2}$	$\frac{-(x^2 + 1)}{2(x^2 + xy - 1)^2}$
4	$-y^2 \sin xy$	$-x^2 \sin(xy)$	$\cos(xy) - xy \sin(xy)$
5	$4ye^{2x+3y}$	$3(2 + 3y)e^{2x+3y}$	$2(1 + 3y)e^{2x+3y}$
6	0	$\frac{8x}{y^3}$	$\frac{-4}{y^2}$
7	$\frac{-24y}{(3x + y)^3}$	$\frac{8x}{(3x + y)^3}$	$\frac{12x - 4y}{(3x + y)^3}$
8	$\frac{2y^4 - 2x^2y^2}{(x^2 + y^2)^2}$	$2 \ln(x^2 + y^2) + \frac{10x^2y^2 + 6y^4}{(x^2 + y^2)^2}$	$\frac{4x^3y}{(x^2 + y^2)^2}$
9	$5v^2e^{uv+2v}$	$5(u + 2)^2e^{uv+2v}$	$5 + 5(uv + 2v + 1)e^{uv+2v}$